Frequency Multiplication Techniques
by Ramon M. Cerda, VP of Engineering, Crystek Corporation

Abstract
This paper will review some of the many ways to achieve frequency multiplication. Frequency multiplication is commonly done in RF/Microwave equipment to generate high-stability, low-noise signals.

How the frequency multiplication is implemented will affect the quality of the final frequency (i.e. phase noise/jitter). What follows is a review of some multipliers, and the reader is encouraged to use the Reference list for more specific design details.

A mathematical review of frequency multiplication will now follow. An ideal signal has no harmonics and can be characterized by

\[ V(t) = V_0 \sin \left( 2\pi v_0 t \right) \]

Eq. 1.1

Where \( V_0 \) = nominal peak voltage
\( v_0 \) = nominal signal frequency

We can expand this ideal signal into a realistic signal by adding amplitude and phase deviation/noise. Eq. 1.1 now becomes

\[ V(t) = V_0 + \varepsilon(t) \sin[2\pi v_0 t + \varphi(t)] \]

Eq. 1.2

Where \( \varepsilon(t) \) = instantaneous amplitude fluctuations
\( \varphi(t) \) = instantaneous phase fluctuations

The main analysis of interest of Eq. 1.2 is frequency-stability which can be derived by taking the time derivative of

\[ \frac{d}{dt} \frac{1}{2\pi} \left[ 2\pi v_0 t + \varphi(t) \right] \]

Which gives the instantaneous frequency:

\[ v(t) = v_0 + \frac{1}{2\pi} \frac{dq(t)}{dt} \]

Eq. 1.3

The difference between the instantaneous frequency \( v(t) \) and nominal frequency \( v_0 \), divided by the nominal frequency is defined as the fractional frequency (or normalized frequency). Hence,

\[ y(t) = \frac{\Delta f}{f} = \frac{v(t) - v_0}{v_0} \]

Eq. 1.4

By definition dividing phase fluctuations by the radiant frequency will give

\[ x(t) = \frac{\varphi(t)}{2\pi v_0} \]

Eq. 1.5

Where \( x(t) \) is called the instantaneous time fluctuations.

Power Spectral Density (PSD)
In order to understand how a signal is altered by multiplication, it will be useful to define \( \Phi(t) \), \( y(t) \) and \( x(t) \) by their power spectral densities (PSD) as listed in Table 1. It represents the power shape of the function. Most of us are very familiar with the PSD of the Gaussian function; a “Bell” curve.

Another useful PSD is the single-sided band (SSB) phase noise to carrier power ratio, \( L(f) \) which has the units of dBc/Hz.

To derive \( L(f) \) in terms of the fractional frequency, we start by multiplying the ideal signal of Eq. 1.1 by \( N \), an integer.

\[ V(t) = V_0 N \sin \left( 2\pi v_0 t \right) \]

Eq. 1.6

The Allen deviation is a two-sample statistics and standard method to describe short-term frequency stability in oscillators. It is the square-root of the Allan Variance. The Allen deviation is used because the normal variance will not converge for large sample sizes of frequency data of oscillators. The Allen deviation is denoted by \( \sigma(t) \).

Frequency Multiplication by a Noiseless Multiplier
Multiplying a signal like Eq. 1.2 by a factor of \( N \) in an ideal noiseless multiplier will affect it as summarized in Table 2.

From Table 2, one can note that the phase noise \( L(f) \) went up by \( 20 \log N \), a well known result. However, \( \Delta f / f \), \( S_y(f) \) and \( \sigma(t) \) are unaffected by frequency multiplication, which may not be well known results. The result that \( \Delta f / f \) stays unchanged is the following: if the original signal has an accuracy of -5.0 ppm, then after multiplication by \( N \), it is still -5.0 ppm.
Step Recovery Diode (SRD) Multiplication
SRDs are very popular for harmonic frequency multiplication and frequency comb generation. SRDs are specially designed diodes where the minority carrier's (electrons on the p side and holes on the n side) lifetime is sufficiently long so as not to recombine; and hence a charge is stored. There are two states of reactance during operation of an SRD. Forward bias corresponds to high capacitance while reversed bias is low capacitance. Due to these two states, an SRD can be considered as a Switching Reactance Multiplier (SRM), or a charge-controlled switch. Figure 1 illustrates a typical SRD multiplier.

Nonlinear Transmission Line Multiplication
Frequency multiplication can also be accomplished with Nonlinear Transmission Lines (NLTL). They are similar to SRD only in the fact that they are both comb generators. Figure 2 is a distributed version using transmission lines and nonlinear capacitors. A simpler implementation uses inductors and varactors as the circuit in Figure 3. Varactors are a clever way to implement nonlinear capacitors since their capacitance will vary nonlinear versus their reverse voltage. This creates a transmission line in which the propagation velocity is voltage dependant.

Direct Multiplication
Direct multiplication is a very efficient method of frequency multiplication. Figure 4 shows the AC circuit representation with two transistor stages. Direct multiplication takes advantage of the nonlinear nature of a saturated collector. In the circuit, L1, C1, C2, L2 and C3 form a bandpass filter. Direct multiplication is usually used to multiply by small integer numbers like, 2, 3, 4 or 5. The bandpass filter and output stage are tuned to the desired integer multiplication.

Mixer Multiplication
Another popular technique for frequency multiplication is to use a mixer as shown in Figure 5. Let

$$VS_1(t) = A_1 \sin(\omega_1 t + \psi_1)$$
$$VS_2(t) = A_2 \sin(\omega_2 t + \psi_2)$$

When $VS_1(t)$ = $VS_2(t)$ then

$$Vout(t) = \frac{1}{2} A_2^2 \cos(2\omega_1 t + 2\psi_1)$$

Hence the frequency and the phase error are doubled.

PLL Multiplication
Phase Locked Loop (PLL) fre-
Frequency multiplication is probably the most popular technique mentioned so far. There are many good books and articles on PLLs. In Figure 6, this particular PLL uses a Voltage Controlled SAW Oscillator (VCSO) for the normal VCO. Sinewave VCSOs can have very low phase noise floors as shown in Figure 7. Because of this, PLL multiplication to a single frequency using a VCSO is a good combination. The PLL will improve the close-in phase noise of the VCSO while maintaining its excellent noise floor.

Low Noise Schottky Diode Odd-Order Multiplier

Figure 8 is an odd-order low noise multiplier. It is a good choice for multiplying crystal oscillators by 3, 5 or 7. When used with low noise Schottky diodes, it will add low excess noise above the theoretical 20logN limit.

References


“Pulse and Waveform Generation with Step Recovery Diodes,” HP note AN 918.

“Harmonic Generation using Step Recovery Diodes and SRD Modules,” HP note 920.
